3. A. A. Samarskii, Introduction to the Theory of Difference Procedures [in Russian], Nauka, Moscow (1974).
4. T. V. Kuskova and L. A. Chudov, "Approximate boundary conditions for a vortex in the calculation of the flow of a viscous incompressible liquid," in: Computation Methods and Programming [in Russian], Computer Center of Moscow State University, No. 11 (1968).

APPROXIMATE SOLUTION OF A PROBLEM OF UNSTEADY CONVECTIVE HEAT EXCHANGE IN LONGITUDINALLY STREAMLINED BUNDLES OF RODS

A. A. Kochubei and A. A. Ryadno UDC 536.24

Unsteady heat exchange is investigated in a longitudinal bundle of rods with a square arrangement during laminar, hydrodynamically stabilized flow of a viscous, incompressible fluid.

In calculations of heat-exchanging systems it is of interest to investigate convective heat exchange in longitudinal flow past bundles of rods. The steady heat exchange in bundles has been studied in [1-3], and elsewhere.

We consider the problem of unsteady heat exchange in a bundle of semi-infinite rods with a square arrangement, with laminar flow of a viscous incompressible fluid. Neglecting the heat of friction and the axial thermal conductivity, assuming the fluid flow to be steady and hydrodynamically stabilized, and the thermophysical properties of the fluid to be constant, we write the energy equation for determination of the temperature field

$$\frac{\partial T}{\partial t}+\omega_z(r,\ \varphi)\frac{\partial T}{\partial z}=a\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2}\right)$$
$$(t>0;\quad z>0;\quad r,\ \varphi\in\Omega),$$

where $T(r, \varphi, z, t)$ is the unknown temperature field; t, time; the z axis is collinear with the direction of the fluid flow; r and φ, cylindrical coordinates; the region Ω:{$0 < \varphi < \pi/4$, $r_0 < r < b/\cos\varphi$} represents a translational element of the bundle, by consideration of which we can limit ourselves by virtue of the symmetry of the rod system (Fig. 1); $\omega_z(r,\varphi)=\frac{\Delta pb^2}{l\mu}\left\{\frac{2}{\pi}\ln\frac{r}{r_0}-\frac{1}{4}\left[\left(\frac{r}{b}\right)^2-\left(\frac{r_0}{b}\right)^2\right]+\sum_{j=1}^{m}\frac{\varepsilon_j}{4j}\left(\frac{r}{b}\right)^{4j}\left[1-\left(\frac{r_0}{r}\right)^{8j}\right]\cos 4j\varphi\right\}$ velocity profile of the fluid flow in a translational element [4].

We assume that at the entrance to the bundle the fluid temperature equals $T_e(r, \varphi, t)$, and at the initial moment ot time $T_i(r, \varphi, z)$. Beginning with $t = 0^+$ the temperature of the external surface of the rod assumes the value $T_S(z, t)$. Hence, for the region under consideration the initial and boundary conditions take the form

$$T(r,\ \varphi,\ z,\ 0)=T_i(r,\ \varphi,\ z);\quad T(r,\ \varphi,\ 0,\ t)=T_e(r,\ \varphi,\ t);$$
$$T(r_0,\ \varphi,\ z,\ t)=T_S(z,\ t);\quad \left.\frac{\partial T}{\partial \nu}\right|_{\Gamma_1\cup\Gamma_2\cup\Gamma_3}=0,$$

where Γ_1:{$r_0 < r \leqslant b$, $\varphi = 0$}; Γ_2:{$r = b/\cos\varphi$, $0 \leqslant \varphi \leqslant \pi/4$}; Γ_3:{$r_0 < r \leqslant \sqrt{2}b$, $\varphi = \pi/4$}; ν is the interior normal to the boundary Ω.

The boundary-value problem in dimensionless form will be

Dnepropetrovsk State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 38, No. 2, pp. 303-308, February, 1980. Original article submitted April 27, 1979.

 0022-0841/80/3802-0192$07.50

$$\frac{\partial\Theta}{\partial\,\mathrm{Fo}}+W(R,\ \varphi)\ \frac{\partial\Theta}{\partial Z}=\frac{\partial^2\Theta}{\partial R^2}+\frac{1}{R}\ \frac{\partial\Theta}{\partial R}+\frac{1}{R^2}\ \frac{\partial^2\Theta}{\partial\varphi^2} \tag{1}$$

$$(\mathrm{Fo}>0;\quad Z>0;\quad R,\ \varphi\in\Omega_1),$$

$$\Theta(R,\ \varphi,\ Z,\ \mathrm{Fo})|_{\mathrm{Fo}=0}=\Theta_i\ (R,\ \varphi,\ Z);$$

$$\Theta(R,\ \varphi,\ Z,\ \mathrm{Fo})|_{Z=0}=\Theta_e(R,\ \varphi,\ \mathrm{Fo});\quad \Theta(R,\ \varphi,\ Z,\ \mathrm{Fo})|_{R=1/\beta}=\Theta_S(Z,\ \mathrm{Fo}); \tag{2}$$

$$\left.\frac{\partial\Theta}{\partial\nu_1}\right|_{\overline{\Gamma}_1\cup\overline{\Gamma}_2\cup\overline{\Gamma}_3}=0.$$

Here Ω_1, ν_1, and $\overline{\Gamma}_i$ (i = 1, 2, 3) are the considered region, the normal to it, and the boundaries Γ_i (i = 1, 2, 3) in dimensionless coordinates.

The problem (1)-(2) will be solved using a method developed in [5, 6] for the investigation of unsteady convective heat exchange for laminar flow of a viscous incompressible fluid in channels of noncircular cross section and based on the joint application of the Bubnov–Galerkin method and the method of characteristics [7].

We seek a solution based on the Bubnov–Galerkin method in the form

$$\Theta_n(R,\ \varphi,\ Z,\ \mathrm{Fo})=\Theta_S(Z,\ \mathrm{Fo})+\sum_{k=1}^{n}a_k(Z,\ \mathrm{Fo})\,\Psi_k(R,\ \varphi),$$

where $\{\Psi_k(R,\ \varphi)\}$ is a system of coordinate functions, satisfying the requirements of the method [8]: the $a_k(Z,\ \mathrm{Fo})$ (k = 1, 2, ..., n) are unknown coefficients — or functions. For the problem being considered, the coordinate functions are chosen by using the method of R-functions [9] in the form

$$\Psi_k=W(R,\ \varphi)\left[X_k-\omega\left(\frac{\partial\omega_3}{\partial R}\ \frac{\partial X_k}{\partial R}+\frac{1}{R^2}\ \frac{\partial\omega_3}{\partial\varphi}\ \frac{\partial X_k}{\partial\varphi}\right)\right],$$

where

$$X_k(R,\ \varphi)=R^{4(k-1)}\cos 4(k-1)\,\varphi;\quad \omega_1=R^2-1/\beta^2;\quad \omega_2=\varphi\,(\pi/4-\varphi);$$

$$\omega_3=1-R\cos\varphi;\quad \omega=\omega_1\omega_2^2+\omega_3-\sqrt{\omega_1^2\omega_2^4+\omega_3^2}.$$

Applying the Bubnov–Galerkin method, for the determination of $a_k(Z,\ \mathrm{Fo})$ we obtain a system of nonlinear partial differential equations

$$\sum_{k=1}^{n}\left(A_{mk}\ \frac{\partial a_k}{\partial\,\mathrm{Fo}}+B_{mk}\ \frac{\partial a_k}{\partial Z}\right)=\sum_{k=1}^{n}D_{mk}a_k+F_m(Z,\ \mathrm{Fo}), \tag{3}$$

where

$$A_{mk}=A_{km}\iint_{\overline{\Omega}_1}\Psi_k\Psi_m d\overline{\Omega}_1;\quad B_{mk}=B_{km}=\iint_{\overline{\Omega}_1}W\Psi_k\Psi_m d\overline{\Omega}_1;$$

$$D_{mk}=D_{km}=-\iint_{\overline{\Omega}_1}\left(\frac{\partial\Psi_k}{\partial R}\ \frac{\partial\Psi_m}{\partial R}+\frac{1}{R^2}\ \frac{\partial\Psi_k}{\partial\varphi}\ \frac{\partial\Psi_m}{\partial\varphi}\right)d\overline{\Omega}_1;$$

$$F_m=-\iint_{\overline{\Omega}_1}\left(\frac{\partial\Theta_S}{\partial\,\mathrm{Fo}}+W\ \frac{\partial\Theta_S}{\partial Z}\right)\Psi_m d\overline{\Omega}_1;$$

$\overline{\Omega}_1$ is the closure of the region Ω, k, m = 1, 2, ..., n.

The uniqueness conditions for system (3) with account of (2) take the form

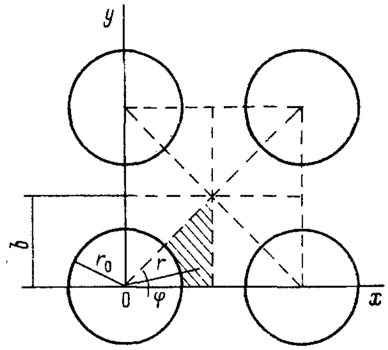

Fig. 1

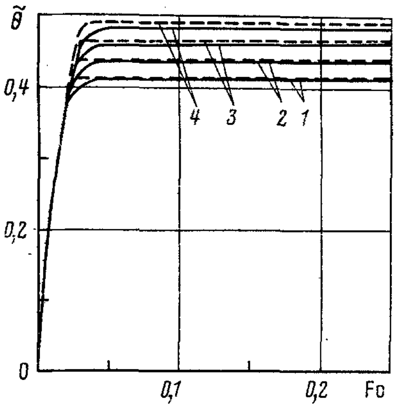

Fig. 2

Fig. 1. Concerning the problem of convective heat exchange in a longitudinal flow past bundles of rods with a square arrangement.

Fig. 2. Variation of the mass-averaged temperature of the fluid: the solid lines are the third approximation; the dashed lines are the second approximation; 1) Z = 0.00075; 2) 0.00085; 3) 0.00095; 4) 0.00105.

$$\sum_{k=1}^{n} A_{mk} a_k (Z,\ 0) = F_m^{\mathrm{i}} (Z); \quad \sum_{k=1}^{n} A_{mk} a_k (0,\ \mathrm{Fo}) = F_m^{\mathrm{e}} (\mathrm{Fo}), \quad m = 1,\ 2,\ \ldots,\ n. \tag{4}$$

Here

$$F_m^{\mathrm{i}} (Z) = \iint\limits_{\Omega_1} [\Theta_{\mathrm{i}} (R,\ \varphi,\ Z) - \Theta_S (Z,\ 0)]\ \Psi_m d\Omega_1;$$

$$F_m^{\mathrm{e}} (\mathrm{Fo}) = \iint\limits_{\bar{\Omega}_1} [\Theta_{\mathrm{e}} (R,\ \varphi,\ \mathrm{Fo}) - \Theta_S (0,\ \mathrm{Fo})]\ \Psi_m d\bar{\Omega}_1, \quad m = 1,\ 2,\ \ldots,\ n.$$

The problem (3)-(4) can be solved in the general case by the method of characteristics just as in [5]. It can easily be seen that the characteristics of system (3) will be straight lines. If T_c = const, $T_e = T_i = T_0$ = const, then the analytical solution of (1)-(2) for n = 2 has the form

$$\Theta_2 (R,\ \varphi,\ Z,\ \mathrm{Fo}) = 1 + \sum_{k=1}^{2} \Psi_k (R,\ \varphi) \begin{cases} a_k^1 (\mathrm{Fo}),\ Z > \mu_1 \mathrm{Fo}, \\ a_k^2 (Z,\ \mathrm{Fo}),\ \mu_2 \mathrm{Fo} < Z < \mu_1 \mathrm{Fo}, \\ a_k^3 (Z),\ Z < \mu_2 \mathrm{Fo}. \end{cases} \tag{5}$$

The expressions for $a_k^1(\mathrm{Fo})$; $a_k^2(Z,\ \mathrm{Fo})$; $a_k^3(Z)$; μ_k, k = 1, 2, aregiven in [6]. The form of solution (5) corresponds to a physical picture of the processes of unsteady convective heat exchange in channels [4].

It is necessary to note that the process of heat exchange being considered occurs in a steady-state regime (in the cross section Z, for example), if the condition Fo > Z/μ_n is satisfied, where the μ_k(k = 1, 2, ..., n) are roots of the equation

$$\begin{vmatrix} B_{11} - \mu_1 A_{11} & B_{12} - \mu_2 A_{12} & \cdots & B_{1n} - \mu_1 A_{1n} \\ B_{21} - \mu_1 A_{21} & B_{22} - \mu_2 A_{22} & \cdots & B_{2n} - \mu_2 A_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ B_{n1} - \mu_1 A_{n1} & B_{n2} - \mu_2 A_{n2} & \cdots & B_{nn} - \mu_n A_{nn} \end{vmatrix} = 0,$$

$$\mu_n = \max_{1 \leqslant k \leqslant n} \{\mu_k\}.$$

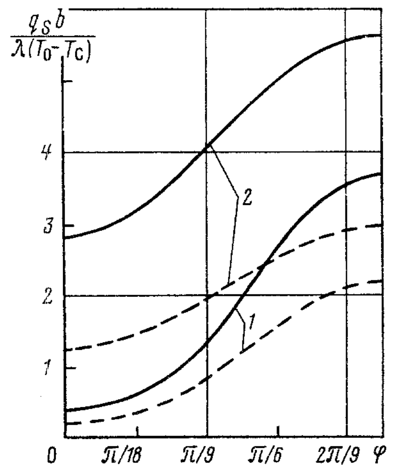

Fig. 3

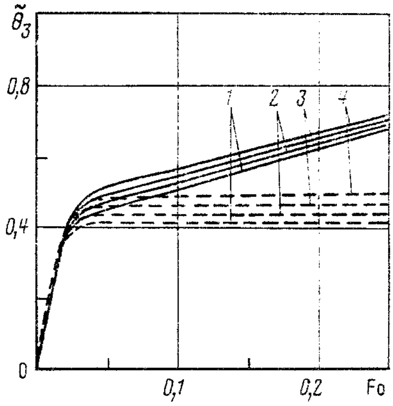

Fig. 4

Fig. 3. Variation of dimensionless thermal flux along the perimeter of the rod: 1) β = 1.05; 2) 1.2.

Fig. 4. Variation of mass-averaged temperature of fluid in the case $\Theta_e = \sin(2Fo)$, $\Theta_i = 0$ (solid lines) and $\Theta_e = 0$, $\Theta_i = 1 - \exp(-Z)$ (dashed lines). The values of Z are the same as in Fig. 2; β = 1.05.

TABLE 1. Values of Coefficients of System (3)-(4) (F_m, F_m^e, F_m^i are calculated for the case $\Theta_e = \Theta_i = 0$, $\Theta_S = 1$)

β	m	$A_{mk}\cdot 10^4$			$B_{mk}\cdot 10^4$		
		k=1	k=2	k=3	k=1	k=2	k=3
1,05	1	1,668	—3,353	6,807	0,083	—0,185	0,413
	2	—3,353	7,771	—18,27	—0,185	0,455	—1,133
	3	6,807	—18,27	49,52	0,413	—1,133	3,138
1,20	1	7,577	—12,03	21,23	0,619	—1,157	2,279
	2	—12,03	25,91	—57,09	—1,157	2,634	—6,167
	3	21,23	—57,09	153,3	2,279	—6,167	16,84
2,00	1	199,6	—135,7	186,7	50,95	—42,77	61,39
	2	—135,7	284,8	—524,5	—42,77	88,98	—170,0
	3	186,7	—524,5	1399	61,39	—170,0	452,4

β	m	$D_{mk}\cdot 10^2$			F_m	$F_m^e = F_m^i$
		k=1	k=2	k=3		
1,05	1	—0,389	0,650	—1,202	0,0	0,00389
	2	0,650	—2,354	7,506	0,0	—0,00650
	3	—1,202	7,506	—32,78	0,0	0,01202
1,20	1	—1,090	1,288	—2,600	0,0	0,01090
	2	1,288	—6,448	21,39	0,0	—0,01288
	3	—2,600	21,39	—98,45	0,0	0,01956
2,00	1	—8,359	4,268	—5,677	0,0	0,08359
	2	4,271	—54,34	154,0	0,0	—0,04271
	3	—5,677	154,0	—803,7	0,0	0,05677

This condition is valid for the solution of the problem in the n-th approximation according to the Bubnov—Galerkin method and, as a particular case, for the second approximation it has the form Fo > Z/μ_2 (the lower line of the solution (5)),

Figure 2 shows the results of calculations of the mass-averaged temperature of the fluid

$$\tilde{\Theta}_n(Z, \mathrm{Fo}) = \frac{\iint\limits_{\bar{\Omega}_1} W(R, \varphi)\, \Theta_n(R, \varphi, Z, \mathrm{Fo})\, d\bar{\Omega}_1}{\iint\limits_{\bar{\Omega}_1} W(R, \varphi)\, d\bar{\Omega}_1}$$

for the case $\Theta_S = 1$, $\Theta_e = \Theta_i = 0$; $\beta = 1.05$ in the second and in the third approximations according to the Bubnov–Galerkin method. It can be seen from the figure that already the second and third approximations of the solution give imperceptible discrepancies in the results, which indicates that the convergence of the proposed method may be practically considered to be rapid. We note that for calculations in the expression for the velocity profile, all terms of the series for which $\varepsilon_j \neq 0$ are taken into account.

Figure 3 shows the variation of the dimensionless thermal flux $q_S b/[\lambda(T_0 - T_c)]$ along the perimeter of the rod, where $q_S = \lambda \partial T/\partial r$ for $R = 1/\beta$. Calculations were carried out for the time Fo = 0.05 in the cross sections of the bundle $Z = 9.5 \cdot 10^{-4}$ (solid lines) and Z = 1 (dashed lines). From the figure it follows that with an increase in β, which is the parameter of the form of the translational element of the bundle, there is also an increase in the density of the thermal flux along the perimeter of the rod, which also corresponds to the physical picture of the process [4].

Fiure 4 represents the results of calculations of the mass-averaged temperature of the fluid (n = 3) for various laws of variation of Θ_e and Θ_i, when $\Theta_S = 1$.

To facilitate calculations of unsteady heat exchange, we present in Table 1 values of the coefficients of the system (3)-(4) for various values of the parameter β (n = 3).

NOTATION

a, thermal conductivity of the fluid; $\Theta = (T - T_0)/(T_c - T_0)$, dimensionless temperature of the fluid; $\mathrm{Fo} = at/b^2$, dimensionless time; $Z = z/(b\mathrm{Pe})$, dimensionless longitudinal coordinate; $\mathrm{Pe} = \bar{\omega} b/a$, Peclet number, where $\bar{\omega} = \Delta p b^2/(l\mu)$; $W = \omega_z/\bar{\omega}$, parameter of the form of the translational element of the bundle; T_c and T_0, certain characteristic temperatures of the wall and of the fluid for t = z = 0, respectively.

LITERATURE CITED

1. V. I. Subbotin, M. Kh. Ibragimov, P. A. Ushakov, V. P. Bobkov, A. V. Zhukov, and Yu. S. Yur'ev, Hydrodynamics and Heat Exchange in Nuclear Energy Installations [in Russian], Atomizdat, Moscow (1975).
2. M. Kh. Ibragimov and A. V. Zhukov, At. Energ., 18, No. 6 (1965).
3. E. M. Sparrow, A. L. Loeffler, Jr., and H. A. Hubbard, J. Heat Transfer, 83, No. 4, 415-422 (1961).
4. B. S. Petukhov, Heat Exchange and Resistance for Laminar Flow of a Fluid in Tubes [in Russian], Énergiya, Moscow (1967).
5. A. A. Kochubei and A. A. Ryadno, Thermal Physics and Heat Engineering, No. 33 (1977).
6. N. M. Belyaev, A. A. Kochubei, and A. A. Ryadno, Teplofiz. Vys. Temp., 15, No. 2 (1977).
7. R. Courant, Partial Differential Equations, Wiley (1962).
8. S. G. Mikhlin, Variational Methods in Mathematical Physics [in Russian], Nauka, Moscow (1970).
9. V. L. Rvachev and A. P. Slesarenko, Algebra of Logic and Integral Transforms in Boundary-Value Problems [in Russian], Naukova Dumka, Kiev (1976).